**Using Benford’s Law and Mean Absolute Deviation to Distinguish Bankrupt Fraudulent Companies and Healthy Currently Operating Companies**

Gary G. Johnson

Anthony W. Varnon

Shuyang Zhang

Southeast Missouri State University

One University Plaza

Cape Girardeau, MO 63701

**Abstract**

This study applies Benford’s Law to four account balances of healthy operating companies and bankrupt fraudulent companies to determine if the probability distributions related to the accounts are different between the two types of companies. Account balances for Revenue, Expense, Income Tax Expense, and Earnings per Share were drawn from three years of quarterly and annual financial statements. Two calculation models of Mean Absolute Deviation were used in assessing the conformity range of the account balances to Benford’s Law. Combination of the two models indicates that the Expense account of the healthy operating companies was in conformity with Benford’s Law, while the Expense account of bankrupt fraudulent companies was in nonconformity. Z scores were calculated to evaluate conformity of each data point to expected occurrences. For both types of companies, significant first-digit nonconformity was found in accounts showing Benford nonconformity. The study shows that Benford’s Law applied to financial statement data can differentiate between accounts in contrasting company types.

**Using Benford’s Law and Mean Absolute Deviation to Distinguish Bankrupt Fraudulent Companies and Healthy Currently Operating Companies**

**Introduction**

In 2002 Enron Corporation filed for bankruptcy after revelation of an accounting fraud. Closely following Enron’s Chapter 11 filing, the United States financial markets were struck by a chain of frauds including Global Crossing Ltd in January, Adelphia Communications in June, MCI WorldCom and Tyco International Ltd in July, Peregrine Systems in September, and Conseco in December. The negative impact of the consecutive financial frauds pushed Congress to enact the Sarbanes-Oxley Act of 2002.

In September 2008, Lehman Brothers Holdings Inc., an investment bank, became the largest ever reported bankruptcy attributed to an accounting fraud. The fall of Lehman Brothers began a worldwide financial and economic crisis.

According to a 2010 study by Beasley, twenty-eight percent of fraudulent companies filed for bankruptcy within two years of experiencing a fraud; the comparable percentage for non-fraudulent companies in the study was thirteen percent. The probability of bankruptcy for a fraudulent company was statistically larger than for a non-fraudulent firm (p-value < 0.001). The study identifies three major fraudulent practices: improper revenue recognition in sixty percent of the frauds; overstated assets either through overvaluing assets or capitalizing expenses in approximately fifty percent; and understatement of expenses and liabilities in thirty percent. Some companies obviously were involved in multiple fraudulent practices. (Beasley, Carcello, and Hermanson, 2010)

Publicly traded companies are required to file accurate and truthful financial reports to the US Security and Exchange Commission (SEC). These reports must be audited by an independent auditor. To detect abnormalities that might signal fraud, auditors have used Benford’s Law. According to this law, leading digits in collections of numbers are likely to be small. Based on this observation, Benford developed probability distributions for numbers 1 through 9 being the first digit. Data sets that do not adhere to these probability distributions may signal accounting errors or fraud.

**Purpose of the Study**

The purpose of this study is to determine if the probability distributions relating to specific account balances are different between bankrupt fraudulent companies and healthy operating public companies. The account balances selected for Benford analysis in this study are Revenue, Expense, Income Tax Expense, and Earnings per Share.

The selection of accounts was influenced by previous research. Deloitte Financial (2008) tracked bankrupt companies and non-bankrupt companies from 2000 to 2007. They found that most fraud was related to two accounts: Revenue and Expenses. Nigrini (2005) determined that Revenue and Earnings per Share numbers were subject to biased management.

**Benford’s Law and Mean Absolute Deviation**

The earliest finding of Benford’s Law was published by Simon Newcomb in the *American Journal of Mathematics* (Newcomb, 1881). The mathematician discovered that the logarithms book in the library was more worn in the front pages and less worn in the back pages. He subsequently reasoned that scientists used tables to look up numbers starting with the numeral one more often than with larger numbers. Newcomb then generated a formula to calculate the probability of a number with any non-zero first digit. The formula is as follows, where d is a number between 1 and 9 and P is the probability:

P (d) = Log10 (1+1/d)

Using this formula, the expected frequencies for digits in first position are included in Table 1.

Table 1: Expected Frequencies of Digits in First Position

|  |  |
| --- | --- |
| Digit | Expected Frequency |
| 1 | .30103 |
| 2 | .17609 |
| 3 | .12494 |
| 4 | .09691 |
| 5 | .07918 |
| 6 | .06695 |
| 7 | .05799 |
| 8 | .05115 |
| 9 | .04576 |

 In the 1930s physicist Frank Benford found the same phenomenon as Newcomb. Benford collected and tested a large set of data containing twenty thousand observations from *Reader’s Digest* articles (Benford, 1938). He found that numbers consistently fell into a pattern with low digits occurring more frequently in the first position than larger digits. He expanded the first digit formula to include expected frequencies for the two combinations of the first and second digits.

The formulas are as follows, where P is probability and Di is the digit sequence of a number:

P (D1 = d1) = log (1+1/d1), d1 ∈ {1, 2…9}

 9

P (D2 = d2) = ∑ log (1+ (1/d1d2)), d2 ∈ {0, 1…9}
 d1=1

P (D1D2 = d1d2) = log (1+1/d1d2)), d1d2 ∈ {10, 11…99}

 Table 2 shows the expected digital frequencies based on these formulas.

Table 2: Expected Digit Frequencies Based on Benford’s Law

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Digit** | **1st Position** | **2nd Position** | **3rd Position** | **4th Position** |
| 0 | n/a | 0.11968 | 0.10178 | 0.10018 |
| 1 | 0.30103 | 0.11389 | 0.10138 | 0.10014 |
| 2 | 0.17609 | 0.19882 | 0.10097 | 0.10010 |
| 3 | 0.12494 | 0.10433 | 0.10057 | 0.10006 |
| 4 | 0.09691 | 0.10031 | 0.10018 | 0.10002 |
| 5 | 0.07918 | 0.09668 | 0.09979 | 0.09998 |
| 6 | 0.06695 | 0.09337 | 0.09940 | 0.09994 |
| 7 | 0.05799 | 0.09035 | 0.09902 | 0.09990 |
| 8 | 0.05115 | 0.08757 | 0.09864 | 0.09986 |
| 9 | 0.04576 | 0.08500 | 0.09827 | 0.09982 |

Nigrini, 1996, “A Taxpayer Compliance Application of Benford’s Law.”

**Benford’s Law Applied to Accounting Data**

Carslaw, 1988 applied Benford’s Law to accounting data. He hypothesized that managers tend to round numbers up when reporting corporate net incomes. Using income data from New Zealand companies to test his hypothesis, he found there were more 0s and fewer 9s in the second digit location than expected by Benford’s Law. He concluded that rounding-up occurred among the New Zealand companies.

Thomas, 1989 published similar findings as Carslaw’s when applying Benford’s Law to U. S. companies. Thomas found that losses reported by U.S. companies presented more nines and fewer zeros in the second digit location. He concluded that U.S. companies were less likely to round up numbers when reporting losses. Additionally, Thomas found that frequencies of five and zero appeared to be much higher in terms of earnings per share numbers, which indicated that EPS were rounded up in U.S. companies. (Thomas, 1989)

**Benford’s Law Used to Detect Fraud**

Mark Nigrini is the first researcher to apply Benford’s law extensively to accounting data aimed at fraud detection. In 1996 Nigrini used Benford’s Law to detect income tax evasion in U.S. individual tax returns (Nigrini, 1996). He defined planned tax evasion as the result of actions to conceal audit trails, while unplanned tax evasion is a blatant manipulation of data by inventing numbers. Results of the study show that lower-income taxpayers practice unplanned tax evasion more than higher-income taxpayers, while higher-income taxpayers are more likely to understate income items and overstate deduction items.

In 1997, Nigrini and Mittermaier examined oil company accounting data for conformity of the digital frequencies to Benford’s Law. (Nigrini and Mittermaier, 1997). The study tested thirty thousand invoices authorized for payment by the accounts payable system. The researcher concluded that the actual frequencies conform to Benford’s Law, while the first two digits deviate from the expected frequencies. In the study Nigrini also describes three conditions under which expected digit frequencies would be valid according to Benford’s Law: Numbers must describe the sizes of similar phenomena; numbers should have no built-in maximums or minimums; numbers are not used to name elements in a data set. (Nigrini and Mittermaier, 1997).

In 2005, Nigrini utilized Benford’s Law to detect changes in earnings management around the Enron fraud (Nigrini, 2005). Results show that both revenue numbers and earnings per share numbers were subject to biased management. Nigrini also examined Enron’s reported numbers between 1997 and 2002. The reports indicate a strong tendency in meeting financial targets.

Similar to Nigrini’s study on earnings management, Johnson, 2005 analyzed reported EPS numbers of various industries and found that quarterly EPS numbers closely conform to Benford’s Law, while companies tend to manage small losses into small gains or report smaller negative earnings per share. (Johnson, 2005)

**Mean Absolute Deviation**

The Mean Absolute Deviation (MAD) is a test recommended to assess the extent of a data set’s conformity to Benford’s Law, which is independent of the size of the data set being considered (Drake and Nigrini, 2000). The higher the MAD, the larger the average difference between the actual and expected proportions (Drake and Nigrini, 2000).

The formula is shown as follows, Where N is the sample size; Xi is the sample value, $\overline{X}$ is the expected value, and fi is the frequency:

 Mean Absolute Deviation =$ \frac{1}{N}\sum\_{1}^{N}fi\left⌊Xi-\overline{X}\right⌋$

The absolute symbol means that the deviation is given a positive sign irrespective of whether it is positive or negative. Individual differences are then totaled and divided by 9 (the number of non-zero leading digits) to yield the mean absolute deviation.

Drake and Nigrini (2000) developed the critical value ranges for the first, second, and first-two digits. In further study, Nigrini pointed out that small data sets may be inclined to false positives errors, when the results conclude nonconformity from unbiased data. (Nigrini, 2012). He adjusted the conformity ranges to increase the effectiveness of the calculation. The adjusted Mean Absolute Deviation critical value ranges are shown in Table 3.

Table 3: Mean Absolute Deviation Critical Value Ranges

|  |  |  |  |
| --- | --- | --- | --- |
| **Conformity Range**  |  **First Digits** | **Second Digits** | **First Two Digits** |
| Close conformity |  0.000-0.006 |  0.000-0.008 |  0.0000-0.0012 |
| Acceptable conformity |  0.006-0.012 |  0.008-0.010 |  0.0012-0.0018 |
| Marginally acceptable conformity |  0.012-0.015 |  0.010-0.012 |  0.0018-0.0022 |
| Nonconformity |  Above 0.015 |  Above 0.012 |  Above 0.0022 |

In the research applying Benford’s law to governmental financial statements, Johnson and Weggenmann (2013) developed an alternative calculation of Mean Absolute Deviation to more effectively address the false positive problem. The Mean Absolute Deviation calculated by Johnson and Weggenmann determined the difference between the actual occurrence rate and the Benford occurrence rate. From these individual differences the mean is subtracted, and the total is divided by 9, the number of possible leading digits. The result is a variated Mean Absolute Deviation. Comparison between the two MAD calculations is exhibited in Table 4.

Table 4: Difference in Calculation of Mean Absolute Deviation

|  |  |
| --- | --- |
| **Drake and Nigrini (2000)** | **Johnson and Weggenmann (2013)** |
| N – Sample size | N – Sample size |
| Xi – Actual occurrence rate | Xi – Difference between actual occurrence rate and Benford occurrence rate |
| X̄ – Benford occurrence rate | X̄ – Mean of the difference between actual occurrence rate and Benford occurrence rate |
| fi – always 1 for this model | fi – always 1 for this model |

**Methodology**

**Selection of Companies**

A representative group for each company type was selected from the largest and most public bankrupt fraudulent companies and the one hundred most trusted public companies in the United States.

**Bankrupt, Fraudulent Companies.** The ten bankrupt fraudulent companies selected for this study met two criteria: They declared bankruptcy between 2001 and 2009; and, according to the SEC, they performed fraudulent financial reporting. Appendix A contains a list of the selected companies, bankruptcy filing dates, and reported fraudulent activity.

**Currently Operating, Healthy Companies.** Forbes magazine publishes an annual list of the one hundred most transparent and trustworthy companies trading on American exchanges. GMI Ratings Services develops the list using Accounting and Governance Risk (AGR) scores ranging from 0 to 100, corresponding to a risk assessment on the quality of corporate accounting and management practices. The AGR score uses an entirely quantitative, statistical process to identify accounting items associated with fraudulent financial statements, as well as governance characteristics associated with firms prosecuted by the [SEC](http://www.businessinsider.com/blackboard/sec) for accounting fraud.

In the trustworthy companies list, the companies were sorted into three groups according to market capitalization: over five billion; between one billion and five billion; and under one billion. Again, ten companies were selected from each market capitalization group based upon the highest AGR score. Appendix B includes a list of the selected companies in each capital category, as well as their AGR scores for 2014.

The importance of looking at market capitalization is suggested by Deloitte Financial (2008). This study found that bankrupt companies with annual revenues of more than $10 billion had an average of approximately 10.8 fraud schemes, while bankrupt companies with annual revenues between $100 million and $10 billion averaged 4.3 schemes.

While the current study’s ratio of operating companies to bankrupt companies is 3 to 1, Deloitte Financial (2008) used a sample with a 5.5 to 1 ratio for non-bankrupt companies to bankrupt companies. The Deloitte study followed 3,438 companies with more than $100 million in revenues, comparing 519 bankrupt companies to a group of 2,919 non-bankrupt companies from 2000 through 2007.

**Data Source and Selection**

The account balances for this study were drawn from three years of quarterly and annual financial statements for each type of company (healthy operating companies and bankrupt fraudulent companies). While datasets containing transactions are more likely to conform to Benford’s Law, several studies have used financial statement data for Benford analysis, including Thomas, 1989; Nigrini, 1989; Johnson, 2005; and Johnson and Weggenmann, 2013.

The SEC database EDGAR (Electronic Data Gathering, Analysis, and Retrieval) is the source for data used in this study. Data consists of account balances for four accounts (Revenue, Expense, Income Tax Expense, and Earnings per Share) from twelve quarters of financial statements for each company, including Form 10-Q quarterly reports and Form 10-K annual reports. The balances extracted from the quarterly financial reports database are comprised of individual distributions, which satisfies the conditions to apply Benford’s Law.

**Data Analysis Methods**

The data for each type of company was subjected to Benford analysis, using MS Excel and ACL Auditing Software. Two Excel spreadsheets were created to summarize data in the twelve quarters’ financial reports. Data were imported into ACL Auditing Software and processed under the command: Analyze using Benford’s Law.

Analysis of each account includes a chart depicting the actual occurrence rate and the Benford occurrence rate, (b) Mean Absolute Deviation results for the level of conformity to Benford’s Law using two approaches, and (c) Z statistic for the numbers in first digit position.

**Findings**

 Findings are organized by the four accounts being examined: Revenue, Expenses, Income Tax Expense, and Earnings per Share. For each account, healthy operating companies will be presented first, followed by bankrupt fraudulent companies.

Findings for each account begin with a chart displaying the companies’ actual distribution of digits in first position and the Benford expected occurrence of each digit in first position. The chart is followed by two Mean Absolute Deviation (MAD) scores (Nigrini and Johnson/Weggenmann), as well as statements of conformity or nonconformity to Benford’s Law based on these scores. Z scores are presented for digits with significant deviations from Benford’s Law.

**Revenue Account Findings**

**Healthy operating companies.** Figure 1 compares healthy operating companies’ first-digit occurrence in the Revenue account balances to Benford’s expected occurrence.

Figure 1: Operating Companies’ Revenue

Using the Nigrini calculation model, the Mean Absolute Deviation is 0.0228. Comparing this MAD to the Critical Value Ranges (Table 3, page 8), the MAD indicates Nonconformity with Benford’s Law. The Johnson and Weggenmann MAD of .0167 also shows Nonconformity.

Z statistic at the 95% confidence level indicates a significant deviation from Benford’s Law for numbers 1 and 4 in the first-digit location. Number 1 occurred 1.18 times more frequently than expected with a difference rate of 5.5%, while number 4 occurred less frequently than expected by a factor of 2.69 to 1 with a difference rate of 6.1%.

**Bankrupt fraudulent companies.** Figure 2 compares bankrupt fraudulent companies’ first-digit occurrence in the Revenue account balances to Benford’s expected occurrence.

Figure 2: Bankrupt Fraudulent Companies’ Revenue

The Nigrini Mean Absolute Deviation is 0.0322, indicating Nonconformity to Benford’s distribution. The Johnson and Weggenmann MAD is .0194, also indicating Nonconformity.

The number 4 deviated significantly from Benford’s Law at the 95% confidence level, occurring 1.92 times more than expected with a differential rate of 9.0%.

**Revenue account summary.** The Revenue account findings show Nonconformity to Benford’s Law by both company types, using both MAD calculation methods.

**Expense Account Findings**

**Healthy operating companies.** Figure 3 compares healthy companies’ first-digit occurrence in the Expense account balances to Benford’s expected occurrences.

Figure 3: Operating Companies’ Expense

The Nigrini Mean Absolute Deviation of 0.0137 indicates Marginally Acceptable conformity to Benford’s Law. Using Johnson and Weggenmann’s calculation, the MAD of .0066 indicates Acceptable Conformity to Benford’s Law.

No significant variation occurs for any number in the first-digit position.

**Bankrupt fraudulent companies.** Figure 4 compares bankrupt fraudulent companies’ first-digit occurrence in the Expense account balances to Benford first-digit expectations.

The Nigrini Mean Absolute Deviation is 0.0391, indicating Nonconformity to Benford’s Law distribution. The Johnson and Weggenmann MAD of .0190 also reflects Nonconformity.

Figure 4: Bankrupt Fraudulent Companies’ Expense

Numbers 4 and 8 vary significantly from Benford’s Law at the 95% confidence level. Number 4 occurred 1.64 times more frequently than expected with a differential rate of 6.2%, while the Number 8 occurred 1.84 times more frequently than expected with a differential rate of 6.1%.

**Expense account summary.** Expense account findings for healthy operating companies showed conformity to Benford’s Law by both calculation methods (Marginally Acceptable and Acceptable Conformity). For bankrupt fraudulent companies, the Expense account was in Nonconformity by both calculation methods.

**Income Tax Expense Findings**

**Healthy operating companies.** Figure 5 compares healthy companies’ first-digit occurrence in the Income Tax Expense account balances to Benford’s expected occurrences.

Figure 5: Operating Companies’ Income Tax Expense

The Nigrini Mean Absolute Deviation is 0.0378, indicating Nonconformity to Benford’s Law distribution. The Johnson and Weggenmann MAD of .0187 also reflect Nonconformity.

Five numbers in the first digit location varied significantly from the Benford expected occurrence rate at the 95% confidence level. Numbers 1, 6 and 9 occurred less frequently than expected with respective differential rates of 8.2%, 3.4%, and 2.4%. Numbers 2 and number 4 occurred more frequently than expected with respective differential rates of 6.3% and 5.3%.

**Bankrupt fraudulent companies.** Figure 6 compares bankrupt fraudulent first-digit occurrence in the Income Tax Expense account balances to Benford’s expected occurrences.

Figure 6: Bankrupt Fraudulent Companies’ Income Tax Expense

The Nigrini Mean Absolute Deviation is 0.0228, showing Nonconformity to Benford’s Law distribution. However, Johnson and Weggenmann’s MAD is 0.0146, showing Marginally Acceptable conformity to the Benford distribution.

None of the Z scores for numbers in the first digit indicate a significant deviation from the expected Benford outcome.

**Income Tax Expense account summary.** The Income Tax Expense account for healthy operating companies is in the Nonconformity range by both MAD calculation models. For the bankrupt fraudulent companies, the account was in Nonconformity by Nigrini and Marginally Acceptable by Johnson and Weggenmann.

**Earnings per Share Findings**

**Healthy operating companies.** Figure 7 compares operating healthy companies’ first-digit occurrence in the Earnings per Share account balance to Benford’s expected occurrences**.**

Figure 7: Operating Companies’ Earnings per Share

The MAD calculated by Nigrini’s method is 0.0193, indicating Nonconformity to Benford’s distribution. MAD using Johnson and Weggenmann’s method is 0.015, indicating Marginally Acceptable conformity.

Numbers 1 and 6 in the leading digit position varied significantly from Benford’s Law expectation at the 95% confidence level. Number 1 occurred less than expected by a factor of 1.30 to 1 with a differential rate of 7.0%. Number 6 occurred 1.54 times more frequently than expected to Benford’s distribution with a differential occurrence rate of 3.6%.

**Bankrupt fraudulent companies.** Figure 8 compares operating bankrupt fraudulent companies’ first-digit occurrence in the Earnings per Share account balances to Benford’s expected occurrences**.**

Figure 8: Bankrupt Fraudulent Companies’ Earnings per Share

The Nigrini Mean Absolute Deviation is 0.0269, indicating Nonconformity. MAD under Johnson and Weggenmann’s method is 0.0143, indicating Marginally Acceptable conformity to Benford’s distribution.

None of the occurrence percentages deviated significantly from Benford’s Law expectation at the 95% confidence level.

**Summary for Earnings per Share account.** For both types of companies, the Earnings per Share account was in Nonconformity by one calculation method and was Marginally Acceptable by the other method.

**Data Summary and Analysis**

 Four accounts were selected to determine if the probability distributions relating to the account balances are different between bankrupt fraudulent companies and healthy operating public companies. The accounts selected are Revenue, Expense, Income Tax Expense, and Earnings per Share.

Data for the four accounts has been presented in eight charts, four charts representing the accounts of the healthy operating companies and four charts representing the accounts of the bankrupt fraudulent companies (Figures 1 through 8, pages 11 through 17). Each chart shows the actual frequency of first digits in the accounts’ balances, in comparison with the expected frequencies of first digits according to Benford’s Law.

Two types of statistical analysis were used to determine an account’s conformity to Benford’s Law: (1) Mean Absolute Deviation (MAD) between actual occurrence of first digits in the account balances and Benford expected occurrence, and (2) Z scores for conformity of individual digits to Benford expectations. The MAD was calculated by two calculation models: Nigrini and Johnson/Weggenmann) (Table 4, page 8). The calculations were then compared to the Critical Value Ranges chart (Table 3, page 8), and a conformity range was assigned for each MAD.

This summary and analysis will first summarize the account data by each type of company, combining the data for the four accounts. The company data will then be combined for final analysis.

**Healthy Operating Companies’ Summary**

Table 5 shows the account findings for the healthy operating companies, including Mean Absolute Deviation scores, Critical Value Ranges, and first-digit nonconformity.

Table 5: Operating Companies’ Conformity to Benford’s Law

|  |  |  |  |
| --- | --- | --- | --- |
| Account | Mean Absolute Deviation (Nigrini) | Mean Absolute Deviation (Johnson-Weggenmann) | First-Digit Nonconformity |
| Revenue | 0.0228Nonconformity | 0.0167Nonconformity | 1, 4 |
| Expense | 0.0137Marginally Acceptable | 0.0066Acceptable Conformity | None |
| Income Tax Expense | 0.0378Nonconformity | 0.0187Nonconformity | 1, 2, 4, 6, 9 |
| Earnings per Share | 0.0193Nonconformity | 0.0150Marginally Acceptable | 1, 6 |

Of the eight Critical Value Ranges shown in the table, five indicate Nonconformity to Benford’s law while three show some level of conformity.

The Nigrini MAD model shows Nonconformity for three accounts and Marginally Acceptable for the other account. The Johnson-Weggenmann model shows two accounts in Nonconformity and two accounts with some level of conformity.

When the models are combined, two of the accounts show some level of conformity to Benford’s Law, while the other two remain in nonconformity. The Expense account shows Marginally Acceptable (Nigrini) and Acceptable Conformity (Johnson-Weggenmann). This account had no digits marked as showing significant difference from the expected occurrence. The Earnings per Share account was Marginally Acceptable (Johnson-Weggenmann), with two nonconforming digits.

Combining the methods had no effect on the other two accounts. Revenue and Income Tax Expense show Nonconformity in both MAD calculations, and Z scores show nonconformity in multiple digits in the first position in both accounts. The Income Tax Expense account had more nonconforming digits than any account throughout the study; i.e., in the datasets of both companies.

**Bankrupt Fraudulent Companies’ Summary**

Table 6 shows the account findings for the bankrupt fraudulent companies, including Mean Absolute Deviation scores, Critical Value Ranges, and first-digit nonconformity.

Table 6: Bankrupt Fraudulent Companies’ Conformity to Benford’s Law

|  |  |  |  |
| --- | --- | --- | --- |
| Account | Mean Absolute Deviation (Nigrini) | Mean Absolute Deviation (Johnson-Weggenmann) |  First-Digit Nonconformity |
| Revenue | 0.0322Nonconformity | 0.0194Nonconformity | 4 |
| Expense | 0.0391Nonconformity | 0.0190Nonconformity | 4, 8 |
| Income Tax Expense | 0.0228Nonconformity | 0.0146Marginally Acceptable | None |
| Earnings per Share | 0.0269Nonconformity | 0.0143Marginally Acceptable | None |

Of the eight Critical Value Ranges shown in the table, six indicate Nonconformity to Benford’s law while two show some level of conformity. As shown in the table, Nigrini calculation shows all four of the accounts in Nonconformity. The Johnson-Weggenmann MADs reinforced the nonconformity of two accounts but added some conformity to two accounts.

Specifically, when the models are combined, Revenue and Expense show Nonconformity by both methods, while Income Tax Expense and Earnings per Share each have one Nonconformity range and one Marginally Acceptable range.

Z scores show significant nonconformity in three first digits of the two nonconforming accounts.

**Comparison of Companies**

**Company comparison by conformity ranges.** The purpose of this study was to determine if the probability distributions relating to specific account balances are different between healthy operating public companies and bankrupt fraudulent companies. Table 7 places the company data side by side to address this purpose.

Table 7: Benford Conformity Ranges, by Company Type

|  |  |  |
| --- | --- | --- |
| Account | Operating Companies | Bankrupt Fraudulent Companies |
| Revenue | 0.0228, Nonconformity (N)0.0167, Nonconformity (J/W) | 0.0322, Nonconformity (N)0.0194, Nonconformity (J/W) |
| Expense | 0.0137, Marginally Acceptable (N)0.0066, Acceptable Conformity (J/W) | 0.0391, Nonconformity (N)0.0190, Nonconformity (J/W) |
| Income Tax Expense | 0.0378, Nonconformity (N)0.0187, Nonconformity (J/W) | 0.0228, Nonconformity (N)0.0146, Marginally Acceptable (J/W) |
| Earnings per Share | 0.0193, Nonconformity (N)0.0150, Marginally Acceptable (J/W) | 0.0269, Nonconformity (N)0.0143, Marginally Acceptable (J/W) |

(N) = Nigrini calculation model

(J/W) = Johnson and Weggenmann calculation model

As shown in Table 7, two accounts differentiate between the two types of companies and two accounts do not differentiate.

Operating companies’ Expense account is in Benford some conformity by both models, while bankrupt fraudulent companies’ Expense account is in nonconformity by both models.

 Income Tax Expense differentiates between company types by only the Johnson-Weggenmann model. The healthy operating companies’ Income Tax Expense account shows Nonconformity by two models, while the Bankrupt Fraudulent account shows some conformity by one model.

The Revenue and Earnings per Share accounts do not differentiate between the companies. The Revenue account shows Nonconformity by both models. Earnings per Share shows one Nonconforming and one Marginally Acceptable result for both types of companies.

**Z Score Comparison.** Z scores further differentiated the accounts of the two types of companies. Table 8 summarizes the nonconforming digits for the two types of companies.

Table 8: Significant First Digits, by Account and Company Type

|  |  |  |
| --- | --- | --- |
| **Account** | **Operating Companies** | **Bankrupt Fraudulent Companies** |
| **Conformity** | **Digits** | **Conformity** | **Digits** |
| Revenue | N, N | 1, 4 | N, N | 4 |
| Expense | MA, AC | None | N, N | 4, 8 |
| Income Tax Expense | N, N | 1, 2, 4,6, 9 | N, MA | None |
| Earnings per Share | N, MA | 1, 6 | N, MA | None |

CC=Close Conformity; AC=Acceptable Conformity; MA=Marginally Acceptable, N=Nonconformity

In the operating companies’ dataset, nine of the thirty-six numbers (four accounts, 9 digits each) in first-digit position vary significantly from Benford’s Law. The Income Tax Expense, an account in the Nonconformity range by both MAD calculations, also shows nonconformity in five of the nine digits at the 95% confidence level.

Among the thirty-six numbers in the first-digit position of the bankrupt fraudulent companies’ dataset, only three digits vary significantly from Benford’s Law. These nonconforming digits were in the Revenue and Expense accounts, two accounts in Nonconformity range by both MAD calculations. No digits vary significantly in accounts with Marginally Acceptable conformity to Benford: Income Tax Expense and Earnings per Share.

**Limitations**

A limitation of this research is the relatively small datasets. The dataset for Operating Companies contains 1440 data points, while the data set for Bankrupt Fraudulent companies contains 480 data points. Although the dataset sizes did result in slightly lower MADs for the larger datasets, the corresponding critical ranges for the two companies show little variation.

Data for Fraudulent Bankrupt Companies may be affected by the “timing of collection” problem, which relates to when in time the issues relating to the bankruptcy occurred relative to when data are collected.

 Another limitation is that account balances collected from financial statements represent aggregated data, rather than transaction level data. Although the large data distributions embedded in an aggregated number will increase the effectiveness of Benford’s Law to identify anomalies, the transaction level data is more helpful in identifying specific accounts where error or fraud may reside.

**Discussion and Conclusion**

**Related Research**

Several studies have applied Benford’s Law to determine abnormalities in accounting data. Some have applied the law to transactions within single accounts for one entity; e.g., Nigrini and Mittermaier, 1997, examined invoices in Accounts Payable for an oil company. Some have applied the law to multiple accounts within one company; e.g., Nigrini, 2005, examined Earnings per Share and Revenue concerning the Enron fraud. Others have applied the law to multiple accounts throughout multiple entities; e.g., Johnson, 2013, applied the law to three accounts in the Comprehensive Annual Financial Reports of the fifty states. Benford’s Law has also been applied to a single account of various industries; e.g., Johnson, 2005, applied the law to the Earnings per Share account in divergent industries.

**Data Analysis**

This study applied two Mean Absolute Deviation (MAD) models and Z scores to each account dataset to determine first-digit conformity to Benford’s Law. Using each account’s Nigrini MAD and Johnson-Weggenmann MAD, the account was then assigned Benford conformity ranges according to Nigrini’s Critical Value Ranges for digits in first position. The MAD results were then compared by company type to determine if the Benford Law analysis differentiated between the accounts of the two types of companies.

**MAD results.** The MAD analysis was in three stages: (1) Nigrini, (2) Johnson-Weggenmann, and (3) combined Nigrini and Johnson-Weggenmann.

The Nigrini MAD model differentiated only one account between the two types of companies. The Expense account of the healthy operating companies was in some conformity with Benford’s Law, while the Expense account of the bankrupt fraudulent companies was in nonconformity. The Nigrini model found all remaining accounts to be in nonconformity--three operating companies’ accounts and all bankrupt fraudulent companies’ accounts.

The Johnson-Weggenmann model, considered a better model for small datasets (Johnson and Weggenmann, 2013), differentiated two accounts: the Expense account and the Income Tax Account. The Expense account of healthy operating companies was in some conformity with Benford, while the Expense account of bankrupt fraudulent companies was in nonconformity. Conversely, the Income Tax Account for healthy operating companies showed nonconformity, while the bankrupt fraudulent companies’ Income Tax Account showed some conformity. The Johnson-Weggenmann model did not differentiate between the Revenue and Earnings per Share accounts, showing both accounts in nonconformity to Benford distribution.

When the two MAD calculations and critical ranges were combined, differentiation was strengthened for the Expense account. The Expense account now showed two conformity ranges for healthy operating companies and two nonconformity ranges for bankrupt fraudulent companies.

Combining the MAD methods also resulted in some differentiation in the Income Tax Account, but in the other direction. The healthy operating companies’ Income Tax Account showed nonconformity to Benford in both models, while the bankrupt fraudulent companies’ account shows some conformity with Johnson-Weggenmann.

Combining the models also confirmed that the Revenue and Earnings per Share accounts did not differentiate between the two companies.

**Z scores.** The total nonconforming digits found in healthy operating companies’ accounts exceeded the total found in bankrupt fraudulent companies’ accounts.

At the account level, the conformity/nonconformity of individual digits supported the MAD conformity ranges within both companies’ datasets. With one exception (Earnings per Share, Operating Companies), the accounts showing some conformity to Benford’s Law have no significant nonconforming first digits, while accounts with both MADs in nonconformity range had one or more significant nonconforming digits. The one account with two conforming MADs (Expense, Operating Companies) had no nonconforming digits, and the account with the most nonconforming digits (Income Tax Expense, Operating Companies) had three MADs in nonconformity range.

**Summary and Future Research**

This study shows that Benford’s Law applied to financial statement data can differentiate between accounts in bankrupt companies and operating companies. In addition, the study supports the use of the Z statistic to differentiate between accounts in contrasting Benford MAD critical ranges. While this study did not lay bare the mystery of the magic bullet, interesting observations were presented to lead to further research refinement to possibly identify different datasets that may prove to be more sensitive to identifying the different Benford distributions between healthy and troubled organizations.

**References**

Beasley, M. S., J. V. Carcello, and D. R. Hermanson. (2010). *Fraudulent Financial Reporting: 1998-2007. An Analysis of U.S. Public Companies.* New York: COSO.

Benford, F. (1938). The law of anomalous numbers. *Proceedings of the American* *Philosophical Society, 78,* 551-572.

Carslaw, C. A. P. N. (1988). Anomalies in income numbers: evidence of goal oriented behavior. *The* *Accounting Review*. LXIII (2):321-327.

Deloitte Financial Advisory Services LLP. (2008). Fraud Detected More Often at Bankrupt Companies.http://www.reuters.com/article/us-bankruptcy-fraud-exclusive-idUSTRE4AN3NJ20081124

Drake, P. D and Nigrini, Mark J. (2000). Computer assisted analytical procedures using Benford’s law*. Journal of Accounting Education* 18, 127-146.

Giroux, Gary. (2008). What went wrong? Accounting fraud and lessons from the recent scandals. *Social Research,* winter 2008, Volume 75, Issue 4, pp. 1205-1238.

Hill, T. (1995). The Significant-Digit phenomenon. *The American Mathematical Monthly*, 102, 322-327.

Johnson, Gary G. (2005). Financial sleuthing using Benford’s Law to analyze quarterly data. *Journal of Forensic Accounting*, Vol. VI, No. 2.

Johnson, Gary G, and Weggenmann, J. (2013). Exploratory research applying Benford’s Law to selected balances in the financial statements of state governments. *Academy of Accounting & Financial Studies Journal,* 17 (3), pp. 31-44.

National Commission on Fraudulent Financial Reporting (NCFFR). (1987). *Report of the National Commission on Fraudulent Financial Reporting*. New York: AICPA.

Nigrini, Mark J. (2005). An assessment of the change in the incidence of earnings management around the Enron-Andersen episode. *Review of Accounting and Finance* 4, 92-110.

Nigrini, Mark J. (2012). *Benford's Law: Applications for Forensic Accounting, Auditing, and Fraud Detection.* John Wiley & Sons, Hoboken, New Jersey. ISSN: 978-1-118-15285-0.

Nigrini, M. J. and L. J. Mittermaier. (1997). The use of Benford’s Law as an aid in analytical procedures. *Auditing: A Journal of Practice & Theory*. 16(2):52-67.

Nigrini, M. J. (1996). Taxpayer compliance application of Benford’s Law. *Journal of the American* *Taxation Association*. 18(1):72-92.

Nigrini, Mark J. (1997). The use of Benford's Law as an aid in analytical procedures. *Auditing: A Journal of Practice & Theory* 16.2: 52-67. <http://www.benfordonline.net/list/byauthor/n>

Saville, AD. (2006). Using Benford's Law to detect data error and fraud: An examination of companies listed on the Johannesburg Stock Exchange. SAJEMS 93 : 341-54

Thomas, J. K. (1989). Unusual patterns in reported earnings. *The Accounting Review*. LXIV (4):773-787.

**Appendix A**

Major Companies That Alleged Fraud and Filed Bankruptcy in 2001-2009

|  |  |  |
| --- | --- | --- |
| Company Name | Filing Date | Financial Fraudulent Reporting Activity |
| Enron Corporation | 2001-12 | Concealed huge debt off balance sheets |
| Global Crossing Ltd | 2002-01 | Network capacity swaps to inflate revenues |
| Tyco International Ltd | 2002-07 | Inflated income by $500 million |
| MCI WorldCom | 2002-07 | Inflated assets by $11 billion |
| Adelphia Communications | 2002-06 | Concealed $250 million of debts |
| Peregrine Systems, Inc | 2002-09 | Overstated sales |
| Conseco Inc | 2002-12 | Failed to make down declined securities |
| Refco Group Ltd | 2005-10 | Concealed bad debts |
| Lehman Brothers Holdings Inc | 2008-09 | Disguised over $50 billion of loans as sales |
| General Motors Corporation | 2009-07 | Misapplication of financial accounting standards |

Appendix B

 Thirty Most Trustworthy Companies by Financial Soundness on 2014 GMI Rating List

|  |  |  |  |
| --- | --- | --- | --- |
| Capital Range | Company Name | Market Capital (In thousands) | AVG AGR score 4 Qtrs |
| Large Capital Companies(> $5 Billion) | Oceaneering International | $7,745 | 93 |
| Rackspace Hosting, Inc | $5,141 | 87 |
| Under Armour Inc | $11,975 | 85 |
| Cabot Oil & Gas Corporation | $14,778 | 84 |
| Tyson Foods, Inc. | $13,554 | 81 |
| Wynn Resorts, Limited | $24,543 | 78 |
| Maxim Integrated Products Inc. | $9,226 | 74 |
| Cintas Corporation | $7,270 | 74 |
| Nordstrom, Inc | $11,888 | 73 |
| Lamar Advertising Co | $5,084 | 72 |
| Mid Capital Companies($1 Billion to $5 Billion) | Casey’s General Stores, Inc. | $2,636 | 99 |
| Tennant Company | $1,132 | 94 |
| Steel Dynamics, Inc. | $3,871 | 93 |
| DSW Inc. | $3,491 | 91 |
| Steelcase Inc. | $1,845 | 91 |
| Con-way Inc. | $2,149 | 89 |
| Greif, Inc. | $1,274 | 89 |
| Knight Transportation | $1,732 | 89 |
| Sun Hydraulics Corporation | $1,115 | 89 |
| Sonoco Products Company | $4,280 | 87 |
| Small Capital Companies($250 Million to $1 Billion) | Altra Industrial Motion Corp | $958 | 97 |
| The Gorman-Rupp Company | $827 | 97 |
| Kimball International Inc | $571 | 95 |
| Universal Electronics Inc | $650 | 95 |
| Comfort Systems USA, Inc. | $618 | 94 |
| Hawkins, Inc. | $378 | 94 |
| Citi Trends, Inc. | $253 | 92 |
| Matrix Service Co | $853 | 92 |
| Shenandoah Telecommunications Co | $635 | 92 |
| Alamo Group, Inc. | $635 | 91 |

Source: <http://www.forbes.com/sites/kathryndill/2014/03/18/americas-100-most-trustworthy-companies/>